UNIT-II

STATICALLY DETERMINATE PINJOINTED PLANE FRAMES

Introduction:

Before discussing the various methods of truss analysis, it would be appropriate to have a brief introduction.

A structure that is composed of a number of bars pin connected at their ends to form a stable framework is called a truss. It is generally assumed that loads and reactions are applied to the truss only at the joints. A truss would typically be composed of triangular elements with the bars on the upper chord under compression and those along the lower chord under tension. Trusses are extensively used for bridges, long span roofs, electric tower, and space structures.

Trusses are statically determinate when the entire bar forces can be determined from the equations of statics alone. Otherwise the truss is statically indeterminate. A truss may be statically (externally) determinate or indeterminate with respect to the reactions (more than 3 or 6 reactions in 2D or 3D problems respectively).

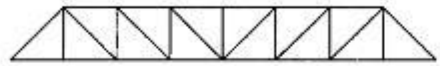
For truss analysis, it is assumed that:

•Bars are pin-connected.
•Joints are frictionless hinges.
•Loads are applied at the joints only.
•Stress in each member is constant along its length.

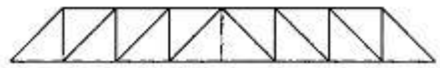
The objective of truss analysis is to determine the reactions and member forces. The methods used for carrying out the analysis with the equations of equilibrium and by considering only parts of the structure through analyzing its free body diagram to solve the unknowns.

Types of trusses:

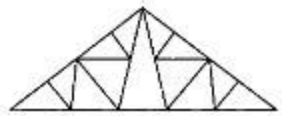
Generally the form selected for a truss depends upon the purpose for which it is required. Examples of different types of truss are shown in Figs (a)-(f); some are named after the railway engineers who invented them. The Pratt, Howe, Warren and K trusses would, for example, be used to support bridge decks and large-span roofing systems (the Howe truss is no longer used for reasons we shall discuss in Section 4.5) whereas the Fink truss would be used to support gable-ended roofs. The Bowstring truss is somewhat of a special case in that if the upper chord members are arranged such that the joints lie on a parabola and the loads, all of equal magnitude, are applied at the upper joints, the internal members carry no load. This result derives from arch theory but is rarely of practical significance since, generally, the loads would be applied to the lower chord joints as in the case of the truss being used to support a bridge deck. Frequently, plane trusses are connected together to form a three-dimensional structure. For example, in the overhead crane shown in Fig, the tower would usually comprise four plane trusses joined ogether to form a 'box' while the jibs would be constructed by connecting three plane trusses ogether to form a triangular cross-section.



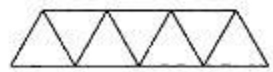
(a) Pratt



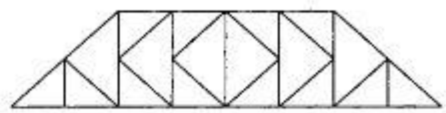
(b) Howe



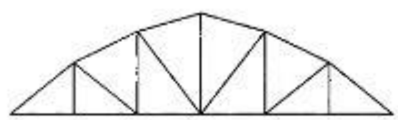
(c) Fink



(d) Warren



(e) K truss



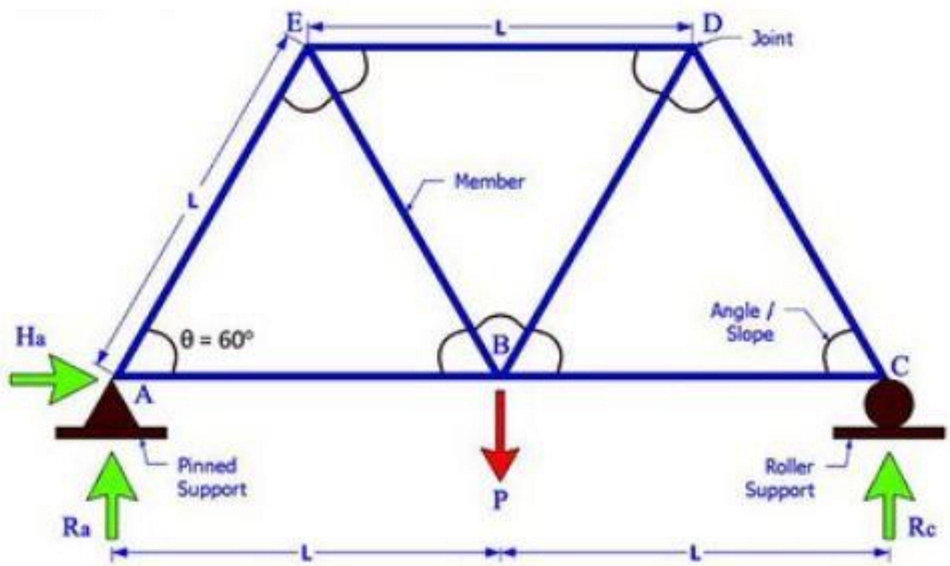
(f) Bowstring

METHOD OF JOINTS:

We start by assuming that all members are in tension reaction. A tension member experiences pull forces at both ends of the bar and usually denoted by positive (+ve) sign. When a member is experiencing a push force at both ends, then the bar is said to be in compression mode and designated as negative (-ve) sign.

In the joints method, a virtual cut is made around a joint and the cut portion is isolated as a Free Body Diagram (FBD). Using the equilibrium equations of $\sum Fx = 0$ and $\sum Fy = 0$, the unknown member forces can be solved. It is assumed that all members are joined together in the form of an ideal pin, and that all forces are in tension (+ve reactions).

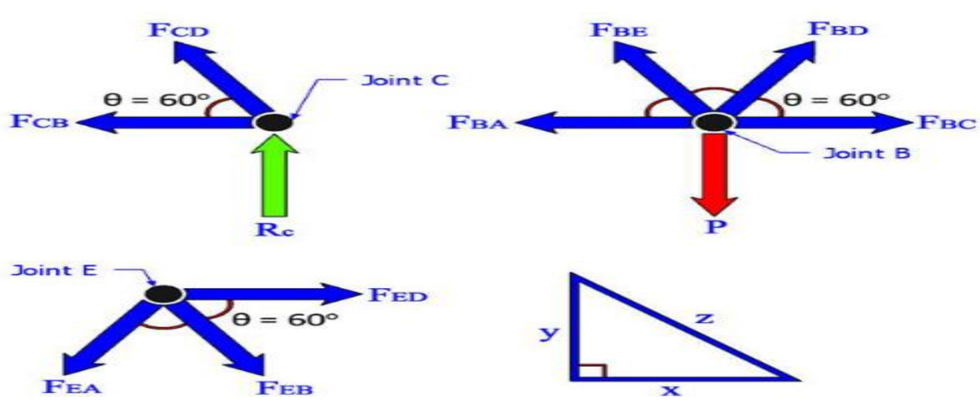
An imaginary section may be completely passed around a joint in a truss. The joint has become a free body in equilibrium under the forces applied to it. The equations $\sum H = 0$ and $\sum V = 0$ may be applied to the joint to determine the unknown forces in members meeting there. It is evident that no more than two unknowns can be determined at a joint with these two equations.



A simple truss model supported by pinned and roller support at its end. Each triangle has the same length, L and it is equilateral where degree of angle, θ is 60° on every angle. The support reactions, Ra and Rc can be determined by taking a point of moment either at point A or point C, whereas Ha = 0 (no other horizontal force).

Here are some simple guidelines for this method:

1. Firstly draw the Free Body Diagram (FBD),
2. Solve the reactions of the given structure,
3. Select a joint with a minimum number of unknown (not more than 2) and analyze it with $\sum$ Fx = 0 and $\sum$ Fy = 0,
4. Proceed to the rest of the joints and again concentrating on joints that have very minimal of unknowns,
5. Check member forces at unused joints with $\sum$ Fx = 0 and $\sum$ Fy = 0,
6. Tabulate the member forces whether it is in tension (+ve) or compression (-ve) reaction.

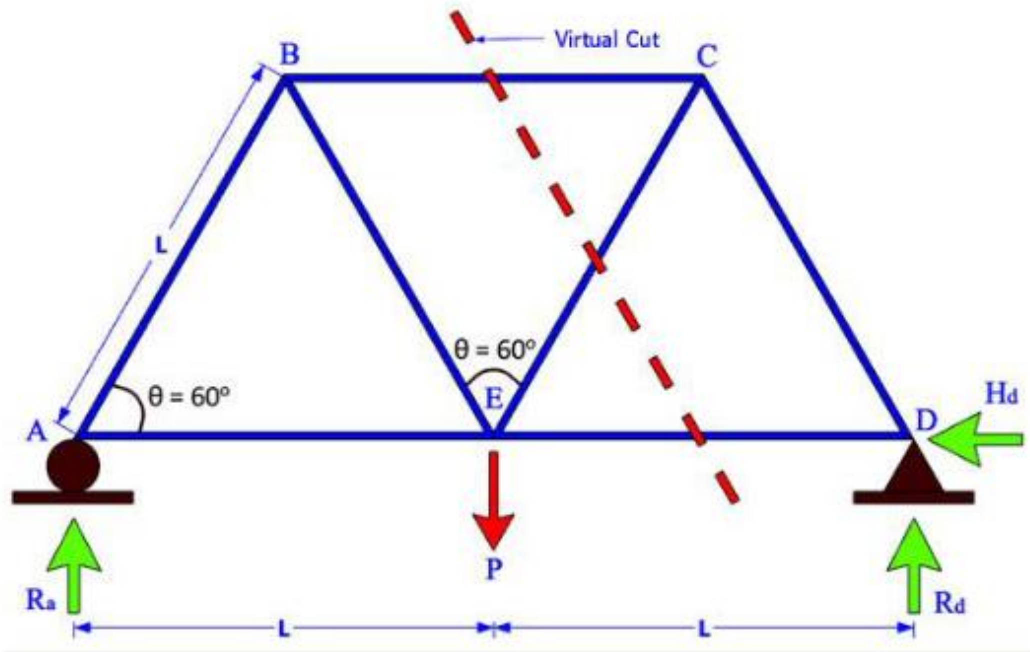


The figure showing 3 selected joints, at B, C, and E. The forces in each member can be determined from any joint or point. The best way to start is by selecting the easiest joint like joint C where the reaction Rc is already obtained and with only 2 unknown, forces of FCB and FCD. Both can be evaluated with $\sum Fx = 0$ and $\sum Fy = 0$ rules. At joint E, there are 3 unknown, forces of FEA, FEB and FED, which may lead to more complex solution compared to 2 unknown values. For checking purposes, joint B is selected to show that the equation of $\sum Fx$ is equal to $\sum Fy$ which leads to zero value, $\sum Fx = \sum Fy = 0$. Each member's condition should be indicated clearly as whether it is in tension (+ve) or in compression (-ve) state.

Trigonometric Functions:
Taking an angle between member x and z...
1.$\cos \theta = x / z$
2.$\sin \theta = y / z$
$\tan \theta = y / x$

METHOD OF SECTIONS:

The section method is an effective method when the forces in all members of a truss are to be determined. If only a few member forces of a truss are needed, the quickest way to find these forces is by the method of sections. In this method, an imaginary cutting line called a section is drawn through a stable and determinate truss. Thus, a section subdivides the truss into two separate parts. Since the entire truss is in equilibrium, any part of it must also be in equilibrium. Either of the two parts of the truss can be considered and the three equations of equilibrium $\sum Fx = 0$, $\sum Fy = 0$, and $\sum M = 0$ can be applied to solve for member forces.

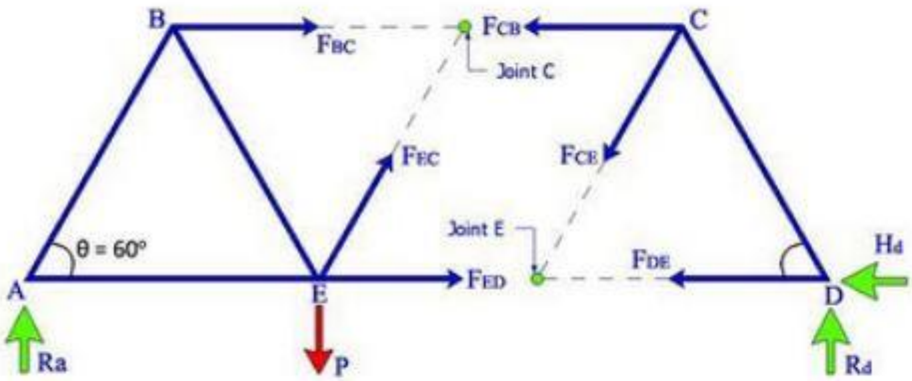


Using the same model of simple truss, the details would be the same as previous figure with different supports profile. Unlike the joint method, here we only interested in finding the value of forces for member BC, EC, and ED.

Few simple guidelines:

Pass a section through a maximum of 3 members of the truss, 1 of which is the desired member where it is dividing the truss into 2 completely separate parts,At 1 part of the truss, take moments about the point (at a joint) where the 2 members intersect and solve for the member force, using $\sum M = 0$,Solve the other 2 unknowns by using the equilibrium equation for forces, using $\sum Fx = 0$ and $\sum Fy = 0$.

Note: The 3 forces cannot be concurrent, or else it cannot be solved.



A virtual cut is introduced through the only required members which is along member BC, EC, and ED. Firstly, the support reactions of Ra and Rd should be determined. Again a good judgment is required to solve this problem where the easiest part would be to consider either the left hand side or the right hand side. Taking moment at joint E (virtual point) clockwise for the whole RHS part would be much easier compared to joint C (the LHS part). Then, either joint D or C can be considered as the point of moment, or else using the joint method to find the member forces for FCB, FCE, and FDE.

Note: Each value of the member's condition should be indicate clearly as whether it is in tension (+ve) or in compression (-ve) state.

Method of Tension Coefficients:

An alternative form of the method of joints which is particularly useful in the analysis of pin-jointed space frames is the *method of tension coefficients*.

Consider the member AB, shown in Fig. 4.19, which connects two pinned joints A and B whose coordinates, referred to arbitrary xy axes, are (x_A, y_A) and (x_B, y_B) respectively; the member carries a *tensile* force, T_{AB}, is of length L_{AB} and is inclined at an angle α to the x axis. The component of T_{AB} parallel to the x axis at A is given by

$$T_{AB}\cos\alpha = T_{AB}\frac{(x_B - x_A)}{L_{AB}} = \frac{T_{AB}}{L_{AB}}(x_B - x_A)$$

Similarly the component of T_{AB} at A parallel to the y axis is

$$T_{AB}\sin\alpha = \frac{T_{AB}}{L_{AB}}(y_B - y_A)$$

We now define a *tension coefficient* $t_{AB} = T_{AB}/L_{AB}$ so that the above components of T_{AB} become:

parallel to the x axis: $t_{AB}(x_B - x_A)$

parallel to the y axis: $t_{AB}(y_B - y_A)$

Equilibrium equations may be written down for each joint in turn in terms of tension coefficients and joint coordinates referred to some convenient axis system. The solution of these equations gives t_{AB}, etc, whence $T_{AB} = t_{AB}L_{AB}$ in which L_{AB}, unless given, may be calculated using Pythagoras' theorem, i.e. $L_{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$. Again the initial assumption of tension in a member results in negative values corresponding to compression. Note the order of suffixes in